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REMOTE SENSING OF CROSSWIND PROFILES USING THE CORRELATION SLOP--ETC(U)

JUL 79 R BARAKAT, T E BUDER DAAD05-76-C-0730

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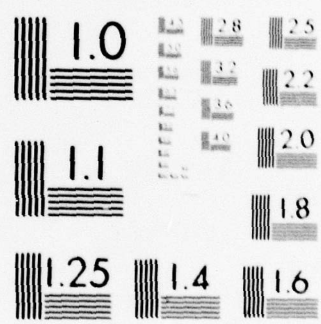

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REMOTE SENSING OF CROSSWIND PROFILES USING THE CORRELATION SLOPE METHOD

Bolt Beranek and Newman Inc.
Cambridge, MA 02138

July 1979

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The correlation slope method relates the slope of the space-time covariance function at zero lag time to the crosswind component along a path via an integral equation of the first kind through the intermediary of a function of the distance between detectors. The integral equation is inverted to find the crosswind profile using the method of singular value decomposition. The effect of noise in the measurement of the correlation slope is considered as well as the influence of finite detector separation. Some numerical results are shown.

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I. INTRODUCTION

There are any number of situations where one would like to obtain information about the behavior of the crosswind along a path. One method of remote sensing that offers promise is that of optical scintillations using correlation techniques.

Provided that the medium is weakly turbulent, then it is possible to write down a nonlinear integral equation between the unknown crosswind profile and the space-time covariance function of the log-amplitude of the incident laser radiation.[1] Lee and Harp[1] have shown how to convert the nonlinear integral equation into a linear integral equation, and working only with the slope at zero time delay of the space-time covariance function of the signal.

Lawrence, Ochs, and Clifford,[2] in an important paper, developed an experimental procedure, utilizing the linear integral equation approach, whereby they measured an averaged crosswind, the average taken with respect to the path-weighting function. Furthermore, they are able to make measurements effectively in real time. In view of their success, it is now an opportune time to consider the inversion problem of reconstructing the crosswind profile itself via measurements of the correlation slope.

The solution of the linear integral equation, Eq. (2) or Eq. (9), is an inverse problem. Inverse problems are known to be ill-posed (i.e., numerically unstable) and any inversion method must be capable of a reasonably robust inversion in the presence of noise.[3,4] We propose to use the method of singular value decomposition to invert the integral equation in the presence of noisy input data.

There have been previous inversion studies of Eq. (2). Peskoff[5] carried out an analytical inversion of the linear integral equation which, although mathematically correct, suffers from the fact that his path must be infinite in order to perform certain of the analytical manipulations. A second attempt to invert the integral equation was

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[1] *R.W. Lee and J.C. Harp, "Weak Scattering in Random Media, With Applications to Remote Probing," Proc. IEEE, 57, 375-406.*

[2] *R.S. Lawrence, G.R. Ochs, and S.F. Clifford, "Use of Scintillations to Measure Average Wind Across a Light Beam," Appl. Opt., 11, 1972, 239-243.*

[3] *M.M. Laventiev, Some Improperly Posed Problems of Mathematical Physics, (Springer-Verlag, New York, 1967).*

[4] *A.N. Tikhonov and V.Y. Aresenin, Solutions of Ill-Posed Problems, (Halsted Press, New York, 1977).*

[5] *A.Peskoff, "Theory for Remote Sensing of Wind-Velocity Profiles," Proc. IEEE, 59, 1971, 324-325.*

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made by Shen,[6] using some naive numerical techniques. Unfortunately, he did not realize that the problem was ill-posed; consequently, his results and conclusions are misleading. A substantial advance was made by Heneghan and Ishimaru[7] who, recognizing that the problem was ill-posed, employed an inversion scheme dependent on statistical regularization.[8] The relation between their method/results and our method/results is briefly discussed in Section IV.

II. PRELIMINARIES

The space time covariance function and the crosswind profile are related to each other by a complicated nonlinear integral equation whose explicit form we need not quote, see Eq. 5 of Ref. 2. Following the suggestion in Ref. 1, the nonlinear integral equation can be made linear (in the velocity profile) by differentiating it with respect to the time covariance function and then setting the time delay equal to zero.

The resulting linear integral equation relating the slope of the space-time covariance function at zero time lag and the crosswind component as a function of the separation between detectors is[2]

$$E(\rho) = \frac{\int_0^L dz C_n^2(z) W(Z,\rho)\, v(z)}{\int_0^L dz C_n^2(z) [z(1-z)]^{5/6}} \tag{1}$$

where z = distance along sight path $(0,L)$
$v(z)$ = crosswind component
$C_n^2(z)$ = refractive index structure coefficient
ρ = distance between detectors
$E(\rho)$ = slope with respect to time of space time covariance function at zero time lag
$W(z,\rho)$ = path-weighting function.

See Fig. 1 for schematic of the geometry. We caution the reader that this expression is valid only in a weakly turbulent medium.

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[6] L.C. Shen, *"Remote Probing of Atmospheric and Wind Velocity By Millimeter Waves," IEEE Trans. Antennas and Prop., AP-18, 1970, 493-497.*

[7] *J.M. Heneghan and A. Ishimaru, "Remote Determination of the Profiles of the Atmospheric Structure Constant and Wind Velocity Along a Line of Sight Path by a Statistical Inversion Procedure," IEEE Trans. Antennas and Prop., AP-22, 1974, 457-464.*

[8] *J.N. Franklin, "Well-Posed Stochastic Extensions of Ill-Posed Linear Problems," J. Math. Anal. Appl., 31, 1970, 682-716.*

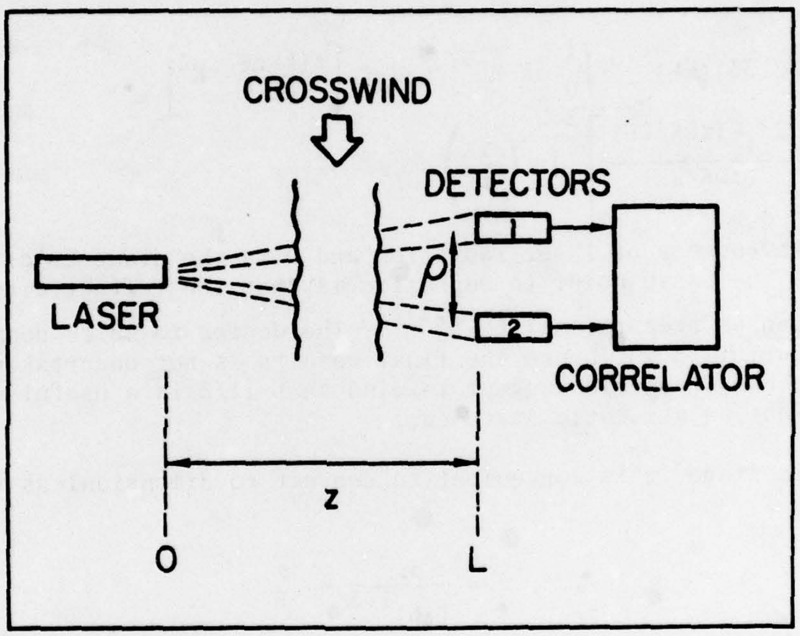

Figure 1. Geometry of bistatic configuration.

We are particularly interested in the path uniform case [$C_n^2(z)$ = constant] characteristic of horizontal propagation.* Equation (1) then becomes

$$E(\rho) = \frac{\int_0^L dzW(z,\rho)v(z)}{\int_0^L dz[z(1-z)]^{5/6}} \quad . \tag{2}$$

The path-weighting function for a spherical wave, assuming that the refractive index size spectrum uses the inertial subrange assumption,[9] takes the form

$$W(z,\rho) \equiv (2.33)(kL)^{5/6} \int_0^\infty dK\, K^{-5/3} \sin^2\left[\frac{z(L-z)}{2kL} K^2\right] \times \left[\frac{2J_1\,(zDK/2L)}{(zDK/2L)}\right]^2 J_1\left(\frac{\rho z K}{L}\right) \tag{3}$$

where k = wavenumber of laser radiation and D is the diameter of the two detectors. The basic point to emphasize is that the refractivity spectrum is taken as proportional to $K^{-11/3}$. The degree to which departures from the power 11/3 influence the final results is not undertaken in this paper, but it should be kept in mind that 11/3 is a useful working number and not an axiomatic statement.

At this stage it is convenient to convert to dimensionless variables:

$$Z \equiv \frac{z}{L} \quad , \quad \beta \equiv \frac{\rho}{(\lambda L)^{1/2}} = \frac{\rho}{F}$$

$$s \equiv K(\lambda L)^{1/2} = KF \quad , \quad \alpha \equiv \frac{D}{(\lambda L)^{1/2}} = \frac{D}{F} \quad . \tag{4}$$

Here

$$F \equiv (\lambda L)^{1/2} \tag{5}$$

is the Fresnel zone number which has the dimensions of a length.

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*See, however, the qualifying remarks in Sec. IV.

[9] V.I. Tatarskii, "The Effects of the Turbulent Atmosphere on Wave Propagation," (US Dept. of Commerce, Springfield, VA, 1971).

The path-weighting function $W(z,\rho)$ can be rewritten as

$$W(Z,\beta) = (2.33)\ (2\pi)^{5/6}\ \frac{F^{7/3}}{\lambda^{5/3}}\ W_1\ (Z,\beta) \tag{6}$$

where $W_1(Z,\beta)$ is dimensionless

$$W_1(Z,\beta) = \int_0^\infty dss^{-5/3}\ \sin^2\left[\frac{Z(1-Z)}{4\pi}\ s^2\right]\left[\frac{2J_1(\alpha Zs/2)}{(\alpha Zs/2)}\right]^2 J_1\ (\beta Zs)\ . \tag{7}$$

The denominator of Eq. 2 can be evaluated in terms of gamma functions

$$\int_0^L dz[z(1-z)]^{5/6} = L^{8/3}\ \frac{\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{11}{6}\right)}{\Gamma\left(\frac{11}{3}\right)}\ . \tag{8}$$

Thus the dimensionless form of Eq. (2) is

$$E(\beta) = \frac{g}{F}\ \int_0^1 dzW_1(Z,\beta)v(Z) \tag{9}$$

where

$$g \equiv \frac{(2.33)(2\pi)^{5/6}\ \Gamma\left(\frac{11}{3}\right)}{\Gamma\left(\frac{11}{6}\right)\ \Gamma\left(\frac{11}{6}\right)} = 48.8938\ . \tag{10}$$

The integral on the right-hand size has dimensions (length/time) since $v(Z)$ is a velocity. Hence, $E(\beta)$ has dimension $(\text{time})^{-1}$ as it should. We caution the reader that $E(\beta)$ and $v(Z)$ must be measured in the same time units. Equation (9) is an integral equation of the first kind for the unknown $v(Z)$ in terms of the measured $E(\beta)$ and known $W(Z,\beta)$.

III. INVERSION VIA SINGULAR VALUES

Given this basic preliminary information, we now pass on to the inversion of Eq. (9) using the method of singular value decomposition.

The integral equation, Eq. (9), was first discretized using N point Simpson's rule

$$\sum_{n=1}^{N} H_n W(\beta_m,Z_n)v(Z_n) = E(\beta_m) \tag{11}$$

where H_n are the weight factors and Z_n the quadrature points. Since we generally want more measured values than reconstruction points, N, we let m = 1, 2, ..., M where $M \geq N$. When Eq. (11) is converted to matrix from, we have

$$\hat{A}\hat{x} = \hat{b} \tag{12}$$

where

$$A_{mn} \equiv H_n W(\beta_m, Z_n)$$

$$x_n \equiv v(Z_n)$$

$$b_m \equiv E(\beta_m)$$

$\hat{A}$ is size M x N (M rows and N columns), $\hat{x}$ is N x 1, and $\hat{b}$ is M x 1. Equation (12) is the basic equation for the inversion.

The matrix A can be written in the following real form (singular value decomposition)

$$\hat{A} = \hat{U} \hat{\Sigma} \hat{V}^+ \tag{13}$$

where $\hat{U}$ is an M x M orthogonal matrix ($\hat{U}\hat{U}^+ = \hat{U}^+\hat{U} = \hat{I}_M$), $\hat{V}$ is an N x N orthogonal matrix. The matrix $\hat{\Sigma}$ is M x N with non-negative elements on the main diagonal and zeros elsewhere

$$\hat{\Sigma} = \left| \begin{array}{cccc} \sigma_1 & & & \\ & \sigma_2 & & \hat{0} \\ & & \ddots & \\ & & & \sigma_N \\ \hline & & \hat{0} & \end{array} \right| \equiv \left| \begin{array}{c} \hat{D} \\ \hat{0} \end{array} \right| . \tag{14}$$

The σ's are termed the <u>singular values</u> of $\hat{A}$ and are the non-negative square roots of the eigenvalues of $\hat{A}^+\hat{A}$ (this is the <u>mathematical</u> definition of the σ's, but they are never evaluated from the definition).

The σ's can be ordered so that $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_N \geq 0$, if rank of $\hat{A} = k\,(\leq N)$, then $\sigma_{k+1} = \sigma_{k=2} = \ldots = \sigma_N = 0$. The columns of $\hat{U}$ are the eigenvectors of $\hat{A}\hat{A}^+$, while the columns of $\hat{V}$ are the eigenvectors of $\hat{A}^+\hat{A}$. Formal proofs of the singular value decomposition of a matrix can be found in Forsyth and Moler,[10] or Lawson and Hanson.[11]

The solution to the minimal least squares problem posed by Eq. (12) is[11]

$$\hat{x} = \hat{A}^{\oplus}\,\hat{b} \tag{15}$$

where $\hat{A}^{\oplus}$ is the pseudoinverse of $\hat{A}$ given by

$$\hat{A}^{\oplus} = \hat{V}\,\hat{\Sigma}^{+}\,\hat{U}^{+} \quad . \tag{16}$$

Here $\hat{\Sigma}^{+}$ is the N x M matrix

$$\hat{\Sigma}^{+} = \begin{vmatrix} \sigma_1^+ & & & 0 \\ & \sigma_2^+ & & \\ & & \ddots & \hat{0} \\ & & & \sigma_N^+ \end{vmatrix} \tag{17}$$

where

$$\sigma_n^+ = \frac{1}{\sigma_n} \quad \text{if} \quad \sigma_n > 0$$

$$= 0 \quad \text{if} \quad \sigma_n = 0 \quad . \tag{18}$$

The solution becomes clearer if the right-hand side of Eq. (15) is written out explicitly.

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[10] *C.B. Moler and G.E. Forsyth, Computer Solution of Linear and Algebraic Systems, (Prentice-Hall, Englewood Cliffs, NJ, 1967).*

[11] *C.L. Lawson and R.J. Hanson, Solving Least Squares Problems, (Prentice-Hall, Englewood Cliffs, NJ, 1974).*

$$\hat{x} = \sum_{n=1}^{k} \frac{\hat{u}_n^{+}\hat{b}}{\sigma_n} \hat{v}_n , \quad k \leq N \tag{19}$$

where $\hat{u}_n$ and $\hat{v}_n$ denote the n^{th} column vectors of $\hat{U}$ and $\hat{V}$ corresponding to σ_n. The sum only runs over $n = 1$ to k, where σ_k is the smallest nonzero singular value. Equation (19) shows that the matrix $\hat{x}$ of rank k ($\leq N$) is a linear combination of k matrices of rank one.

The ill-posed nature of the inversion is directly evident. The smaller singular values entering into the denominator tend to greatly magnify any error in the measured data vector $\hat{b}$, thereby resulting in a spurious solution. To alleviate this, the expansion is terminated before the contamination due to the numerically small singular values sets in. One way to achieve this, Blake and Barakat,[12] is to to set

$$\sigma_n^{+} = \frac{1}{\sigma_n} \quad \text{if} \quad \sigma_n > \varepsilon$$

$$= 0 \quad \text{if} \quad \sigma_n < \varepsilon \tag{20}$$

where the criterion for picking ε is

$$\frac{\varepsilon}{\sigma_0} >> \text{noise.} \tag{21}$$

Physically this procedure has the effect of ignoring high frequency components of $\hat{b}$ which are the main cause of the numerical instability. Nevertheless this procedure entails a somewhat arbitrary judgment as to when to terminate the summation. Unfortunately so do any other criteria.

An added advantage to the use of singular values is that the singular values are very stable to perturbations in the matrix elements, in that perturbations of the matrix elements produce perturbations in the singular values of the same order of magnitude.

In applying the singular value decomposition to our problem, we used the algorithm described in Reference 11.

Given that we have an inversion algorithm we must now consider the following problem. The distance β between detectors in Eq. (9) runs over the range $(0,\infty)$. In order to effect the inversion (by singular value decomposition or any other method), one must let the spacing

[12] *J. Blake and R. Barakat, "The Inversion Problem for Two Fold Photoelectron Counting Statistics," Cand. J. Phys., 53, 1975, 1215-1220.*

between detectors become infinite, at least in principle! Obviously this cannot be done experimentally so that the question to be answered is: "What is the acceptable upper limit on β (call it β_{max}) for which the inversion works in the noiseless case?"

In order to answer this question, we first consider the direct problem: given the crosswind profile determine $E(\beta)$. This is simply the integration of $v(Z)$ over the path with respect to the path-weighting function $W(Z,\beta)$.

Before continuing these calculations, we choose representative values of L, D, and λ

$$L = 1.5 \cdot 10^5 \text{ cm} = 1.5 \text{ km}$$

$$D = 1.27 \text{ cm}$$

$$\lambda = 6.33 \cdot 10^{-5} \text{ cm} \quad .$$

Based on these values we have

$$\alpha = 0.412$$

$$F = 3.08 \text{ cm} \quad .$$

These values will be used for all numerical calculations in this paper.

The behavior of the path-weighting function $W(Z,\beta)$ is shown in Fig. 2. Evaluation of $W(Z,\beta)$ was by high order quadrature sufficient to guarantee four-digit accuracy. As the detector spacing β is increased, W selectively accentuates various parts of the optical path with large spacings concentrating on the end near the laser. Incidentally, in a working system the detectors would remain fixed and the apparent detector separation would be done by scaling the optics.

We employed

$$v(Z) = 650\, e^{-Z} \cos(2\pi Z) \text{ cm/sec} \tag{22}$$

$$= 650\, e^{-Z} \cos(4\pi Z) \text{ cm/sec} \tag{23}$$

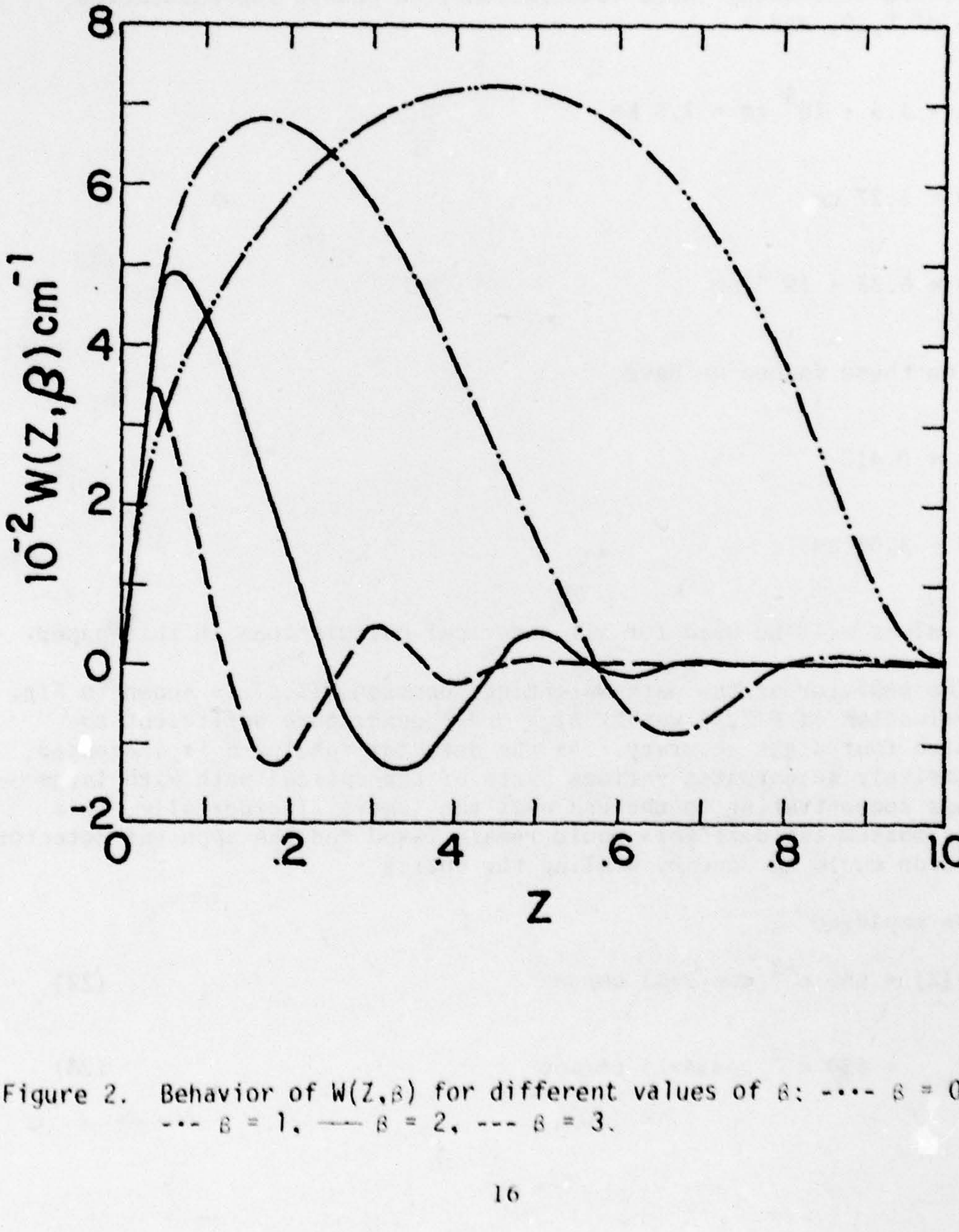

Figure 2. Behavior of $W(Z,\beta)$ for different values of β: ---- β = 0.3, --- β = 1, —— β = 2, --- β = 3.

as test crosswind profiles. Recall that 1 knot ≈ 50 cm/sec. The resultant shapes of the slope of the space-time covariance function at zero time lag as a function of detector separation β, $E(\beta)$, are shown in Fig. 3. Needless to say they display a complicated behavior for small β, but then tend slowly to zero in monotone fashion for large β. Large number of sign changes in $v(Z)$ manifest themselves in a more complicated behavior of $E(\beta)$ for small β. Several other crosswind profiles were run: they all indicate that ultimately $E(\beta)$ tends to zero as β is increased.

Based on this information, we set $\beta_{max} = 3$ and $\beta_{max} = 4$, then inverted the profile given by Eq. (22) using a uniform spacing of $\Delta\beta = 0.1$ $(M = 31)$ and requiring $N = 21$. The inversions are shown in Fig. 4 for $\beta_{max} = 3$ (open circles) and $\beta_{max} = 4$ (solid circles). Both yield excellent fits to the profile for $Z \geq 0.4$; however for $Z < 0.4$ the inversion points for $\beta_{max} = 3$ oscillate about the true profiles whereas those for $\beta_{max} = 4$ are still very good. It is an artifact of singular value decomposition that the inversion points for $Z = 0, 1$ are always zero. There are, of course, 21 singular values since $N = 21$. Values of $\beta_{max} > 4$ did not yield reconstruction significantly better than those for $\beta_{max} = 4$. On the other hand, profiles inverted for $\beta_{max} < 3$ were almost useless due to wild oscillations. At least with respect to the profiles we inverted, it appears that $\beta_{max} = 4$ is a reasonable compromise between accuracy and physical realizability.

Even in the "noiseless" case we do not employ all 21 singular values in the reconstruction; rather we employ 19 singular values because σ_{21} and σ_{20} are enough in error to cause serious distortions in the reconstruction of the crosswind profile.

In order to mimic the experimental situation, we add signal dependent noise to $E(\beta)$ in the following fashion:

$$E(\beta)\Big|_{noisy} = (1 + \delta\mu)E(\beta)\Big|_{noiseless} \quad . \tag{24}$$

Here δ is a small number and μ is a random variable governed by a rectangular probability density function

$$f(\mu) = \frac{1}{2} \, , \qquad -1 < \mu < 1$$

$$= 0 \, , \quad \text{elsewhere} \quad . \tag{25}$$

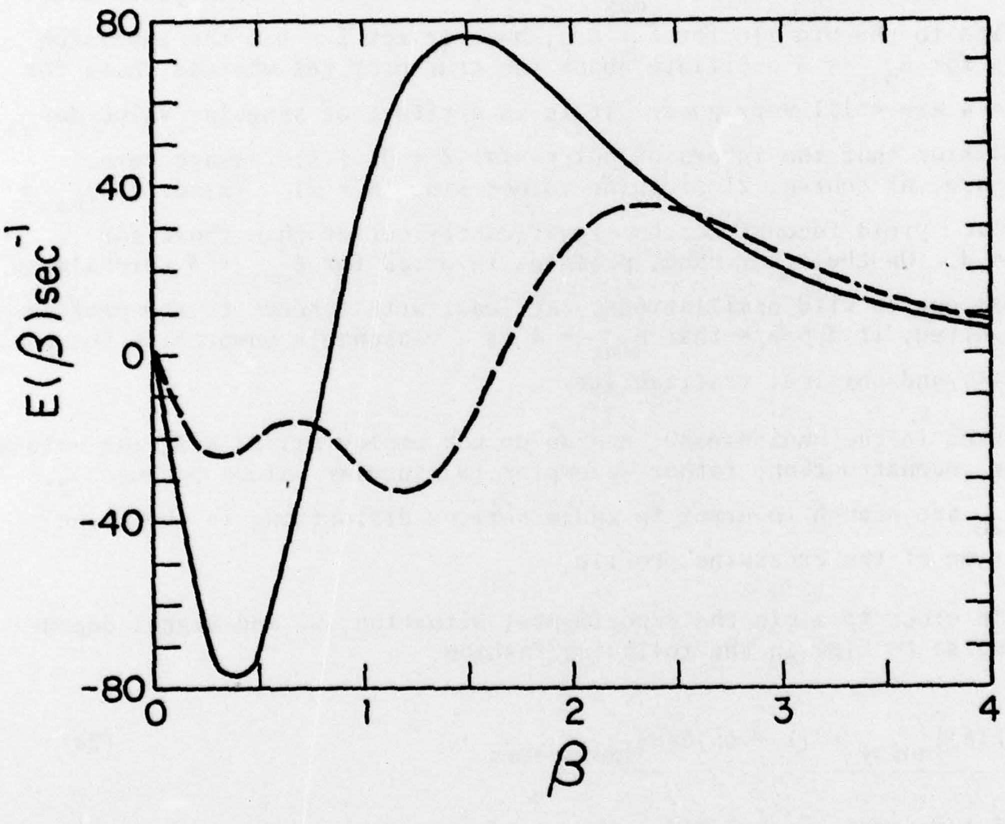

Figure 3. Slope at zero time delay, E(β), as a function of detector separation β for crosswind profile given by Eq. (22) (solid line) and by Eq. (23) (dotted line).

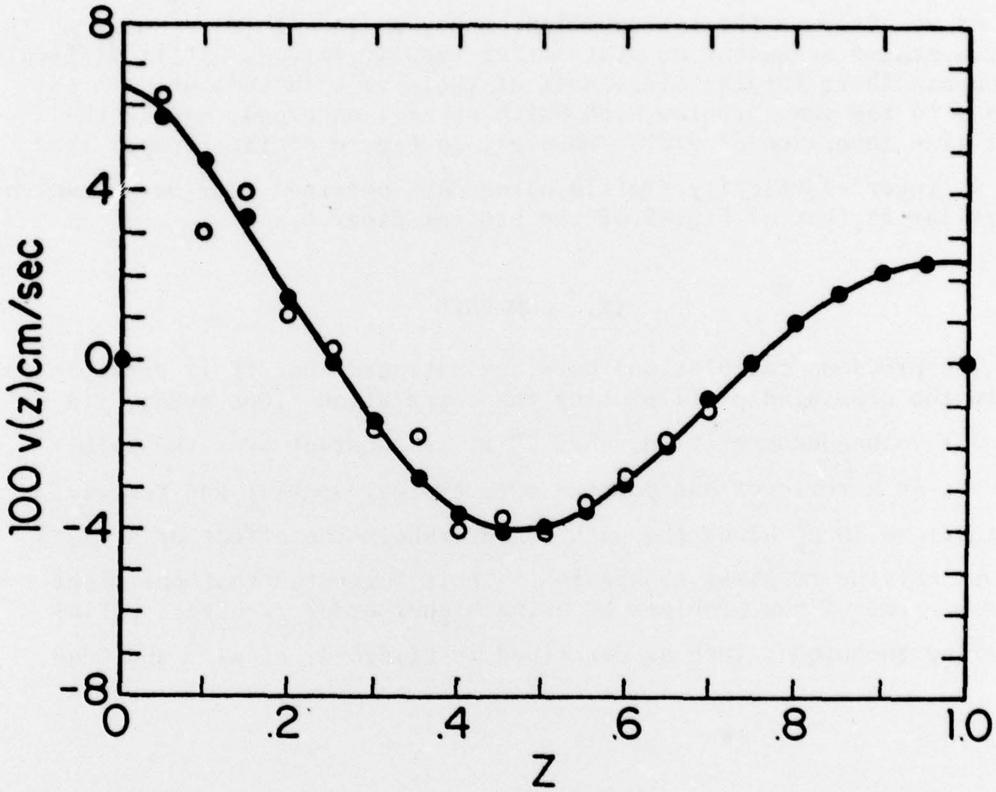

Figure 4. Inversion of v(z) corresponding to Eq. (22) by singular value decomposition using 19 σ's in the noiseless situation: solid is actual profile, open circles correspond to $\beta_{max} = 3$ and solid circles to $\beta_{max} = 4$.

A typical inversion realization of the profile given by Eq. (22) is shown in Fig. 5. The amount of "noise" added to $E(\beta)$ via Eq. (24) was 5% (i.e., $\delta = .025$). According to the recipe quoted in Eq. (21), this means that we should use 16 singular values in the reconstruction of the profile.

A second profile, Eq. (23), was also reconstructed and the results are shown in Fig. 6. The solid circles are the reconstructed values of the profile in the noiseless case with $\beta = 4$. The solid triangles are the reconstructed values of a typical sample realization with 3% noise (i.e., $\delta = .015$) added to $E(\beta)$. The results speak for themselves.

As we noted in the introduction, Heneghan and Ishimaru[7] used an inversion scheme dependent on statistical regularization. It is difficult to compare their results since most of their calculations are not addressed to the same problem with which we are concerned, namely the point-wise inversion of $v(Z)$. However, in Fig. 6 of their paper they show an inverted velocity profile using data obtained from Harp[13] which is similar to that of Fig. 5 of the present paper.

IV. COMMENTS

The previous calculations have demonstrated that it is possible to obtain the crosswind profile using the correlation slope method via singular value decomposition, when $C_n^2(z)$ is constant over the path. However, as a reviewer has pointed out, typical spatial and temporal fluctuations in C_n^2 along the path can overwhelm the effect of $W(z,\rho)$ by just varying receiver separation. It is suggested that one might overcome some of the problems by using higher order receiver spatial filtering techniques such as described in Clifford, et al[14] and Ochs,

[13] *J.C. Harp, "A Line-of-Sight Propagation Experiment for Resolving the Motions and Turbulent Structure of the Atmosphere," Sci. Report No. 1, Radioscience Lab, Stanford Univ., SEL-71-042, Aug. 1971.*

[14] *S.F. Clifford, G.R. Ochs and T. Wang, "Optical Wind Sensing by Observing the Scintillations of a Random Scene," Appl. Opt., 14, 1975, 2844-2850.*

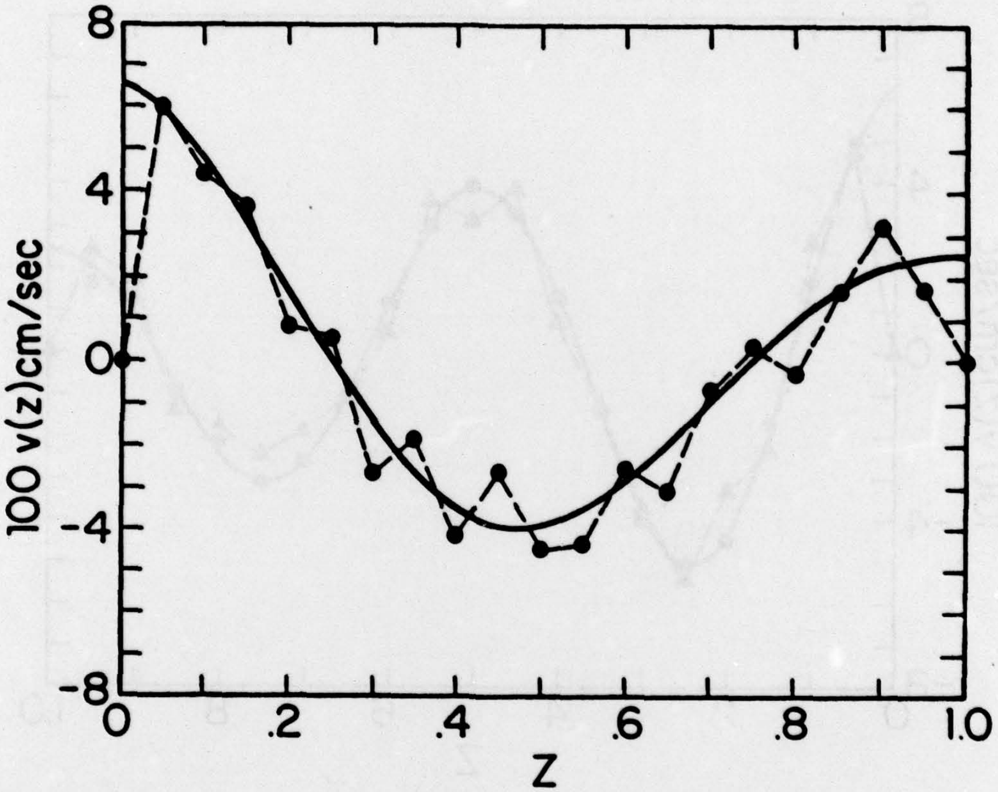

Figure 5. Inversion of v(z) corresponding to Eq. (22) using 16 σ's 4% noise in E(β), β_{max} = 4.

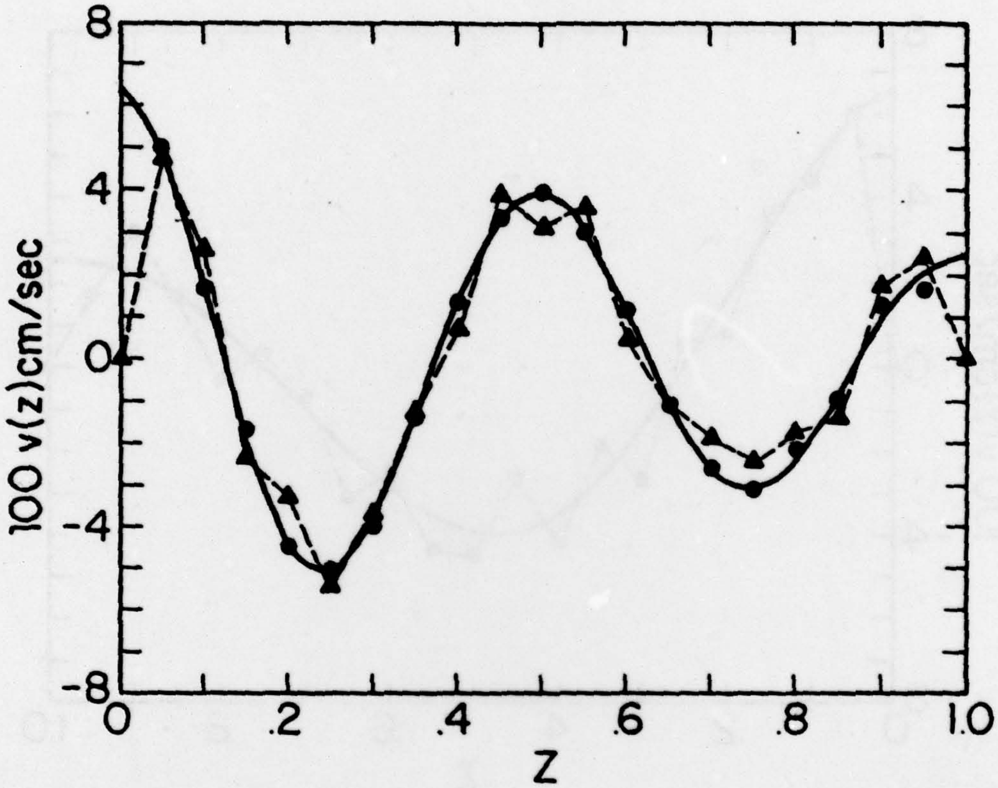

Figure 6. Inversion of v(Z) corresponding to Eq. (23). Solid line is actual profile, solid circles correspond to noiseless situation with $\beta_{max} = 4$, solid triangles correspond to 3% in $E(\beta)$ with $\beta_{max} = 4$.

et al,[15] Lee.[16]. Such an analysis has been initiated by one of us (RB). See also Leuenberger, et al.[17]

It is tempting, when $C_n^2(z)$ varies over the path, to invert the integral equation relating to C_n^2 to the log-amplitude of the covariance for zero time delay using the inversion scheme (e.g., Reference 18) which will guarantee $C_n^2 \geq 0$. The resultant C_n^2 could then be substituted into Eq. (1) for the crosswind profile. Unfortunately, this approach will fail because, as Strohbehn[19] has shown, one cannot distinguish between a C_n^2 layer and a change in the turbulence spectral power law.

ACKNOWLEDGEMENTS

We are indebted to Dr. Paul Deitz of BRL for his continued encouragement and help during the course of this investigation. R. Barakat was supported under Contract DAAD05-76-C-0730 with the Ballistic Research Laboratory, APG, MD.

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[15] *G.R. Ochs, T. Wang, R.S. Lawrence and S.F. Clifford, "Refractive-Turbulence Profiles Measured by One-Dimensional Spatial Filtering of Scintillations," Appl. Opt. 15, 1976, 2504-2510.*

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